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Scattering of electromagnetic waves by many thin cylinders

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Abstract

Electromagnetic wave scattering by many parallel to $z-$axis, thin, perfectly conducting, circular infinite cylinders is studied asymptotically as $a \to 0$. Let D_m be the crossection of the $m-$th cylinder, a be its radius, and $\hat{x}_m = (x_{m1}, x_{m2})$ be its center, $1 \leq m \leq M$, $M = M(a)$. It is assumed that the points $\hat{x}_m$ are distributed so that

$$\mathcal{N}(\Delta) = \ln \frac{1}{a} \int_{\Delta} N(x) dx [1 + o(1)],$$

where $\mathcal{N}(\Delta)$ is the number of points $\hat{x}_m$ in an arbitrary open subset Δ of the plane xoy. The function $N(x) \geq 0$ is a given continuous function. An equation for the self-consistent (efficient) field is derived as $a \to 0$. The cylinders are assumed perfectly conducting. A formula is derived for the effective refraction coefficient in the medium in which many cylinders are distributed. These cylinders may model nanowires embedded in the medium. Our result shows how these cylinders influence the refraction coefficient of the medium.

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1 Introduction

There is a large literature on electromagnetic (EM) wave scattering by an array of parallel cylinders (see, e.g., [2], where there are many references given, and [3]). Electromagnetic wave scattering by many parallel to $z-$axis, thin, perfectly conducting, circular, of radius a, infinite cylinders is studied in this paper asymptotically as $a \to 0$. The cylinders are thin in the sense $ka \ll 1$, where k is the wave number in the exterior of the cylinders,

The novel points in this paper include:

1) The solution to the wave scattering problem is considered in the limit $a \to 0$ when the number $M = M(a)$ of the cylinders tends to infinity at a

suitable rate. The equation for the limiting (as $a \to 0$) effective (self-consistent) field in the medium is derived,

2) This theory is a basis for a method for changing refraction coefficient in a medium. The thin cylinders model nanowires embedded in the medium. The basic physical resul of this paper is formula (48), which shows how the embedded thin cylinders change the refraction coefficient $n^2(x)$.

Some extension of the author's results ([7]-[13]) is obtained for EM wave scattering by many thin perfectly conducting cylinders.

Let D_m, $1 \leq m \leq M$, be a set of non-intersecting domains on a plane P, which is xoy plane. Let $\hat{x}_m \in D_m$, $\hat{x}_m = (x_{m1}, x_{m2})$, be a point inside D_m and C_m be the cylinder with the cross-section D_m and the axis, parallel to z-axis, passing through $\hat{x}_m$. We assume that $\hat{x}_m$ is the center of the disc D_m if D_m is a disc of radius a.

Let us assume that the cylinders are perfect conductors. Let $a = 0.5 \mathrm{diam} D_m$. Our basic assumptions are

$$ka \ll 1, \tag{1}$$

where k is the wave number in the region exterior to the union of the cylinders, and

$$\mathcal{N}(\Delta) = \ln \frac{1}{a} \int_\Delta N(\hat{x}) d\hat{x} [1 + o(1)], \quad a \to 0, \tag{2}$$

where $\mathcal{N}(\Delta) = \sum_{\hat{x}_m \in \Delta} 1$ is the number of the cylinders in an arbitrary open subset of the plane P, $N(\hat{x}) \geq 0$ is a continuous function, which can be chosen as we wish. The points $\hat{x}_m$ are distributed in an arbitrary large but fixed bounded domain on the plane P. We denote by Ω the union of domains D_m, by Ω' its complement in P, and by D' the complement of D in P. The complement in $\mathbb{R}^3$ of the union C of the cylinders C_m we denote by C'.

The EM wave scattering problem consists of finding the solution to Maxwell's equations

$$\nabla \times E = i\omega\mu H, \tag{3}$$

$$\nabla \times H = -i\omega\epsilon E, \tag{4}$$

in C', such that

$$E_t = 0 \text{ on } \partial C, \tag{5}$$

where ∂C is the union of the surfaces of the cylinders C_m, E_t is the tangential component of E, μ and ϵ are constants in C', ω is the frequency, $k^2 = \omega^2 \epsilon\mu$, k is the wave number. Denote by $n_0^2 = \epsilon\mu$, so $k^2 = \omega^2 n_0^2$. The solution to (3)-(5) must have the following form

$$E(x) = E_0(x) + v(x), \quad x = (x_1, x_2, x_3) = (x, y, z) = (\hat{x}, z), \tag{6}$$

where $E_0(x)$ is the incident field, and v is the scattered field satisfying the radiation condition

$$\sqrt{r} \left(\frac{\partial v}{\partial r} - ikv \right) = o(1), \quad r = (x_1^2 + x_2^2)^{1/2}, \tag{7}$$

and we assume that

$$E_0(x) = k^{-1} e^{i\kappa y + ik_3 z}(-k_3 e_2 + \kappa e_3), \quad \kappa^2 + k_3^2 = k^2, \tag{8}$$

$\{e_j\}$, $j = 1, 2, 3$, are the unit vectors along the Cartesian coordinate axes x, y, z. We consider EM waves with $H_3 := H_z = 0$, i.e., E-waves, or TH waves,

$$E = \sum_{j=1}^{3} E_j e_j, \quad H = H_1 e_1 + H_2 e_2 = \frac{\nabla \times E}{i\omega\mu}. \tag{9}$$

One can prove (see Appendix) that the components of E can be expressed by the formulas:

$$E_j = \frac{ik_3}{\kappa^2} U_{x_j} e^{ik_3 z}, \quad j = 1, 2, \quad E_3 = U e^{ik_3 z}, \quad U = \frac{\kappa}{k} u, \tag{10}$$

where $u_{x_j} := \frac{\partial u}{\partial x_j}$, $u = u(x, y)$ solves the problem

$$(\Delta_2 + \kappa^2) u = 0 \text{ in } \Omega' \tag{11}$$

$$u|_{\partial\Omega} = 0, \tag{12}$$

$$u = e^{i\kappa y} + w, \tag{13}$$

and w satisfies the radiation condition (7). Similar calculations are done with Borgnis potentials (see, e.g., [1]). The unique solution to (3)-(8) is given by the formulas:

$$E_1 = \frac{ik_3}{\kappa^2} U_x e^{ik_3 z}, \quad E_2 = \frac{ik_3}{\kappa^2} U_y e^{ik_3 z}, \quad E_3 = U e^{ik_3 z}, \tag{14}$$

$$H_1 = \frac{k^2}{i\omega\mu\kappa^2} U_y e^{ik_3 z}, \quad H_2 = -\frac{k^2}{i\omega\mu\kappa^2} U_x e^{ik_3 z}, \quad H_3 = 0, \tag{15}$$

where $U_x := \frac{\partial U}{\partial x}$, U_y is defined similarly, and $u = u(\hat{x}) = u(x, y)$ solves scalar two-dimensional problem (11)-(13). These formulas are derived in the Appendix for convenience of the reader.

Problem (11)-(13) has a unique solution (see, e.g., [4]).

Our goal is to derive an asymptotic formula for this solution as $a \to 0$. Our results include formulas for the solution to the scattering problem, derivation of the equation for the effective field in the medium obtained by embedding many thin perfectly conducting cylinders, and a formula for the refraction coefficient in this limiting medium. This formula shows that by choosing suitable distribution of the cylinders, one can change the refraction coefficient, one can make it smaller than the original one.

The paper is organized as follows.

In Section 2 we derive an asymptotic formula for the solution to (11)-(13) when $M = 1$, i.e., for scattering by one cylinder.

In Section 3 we derive a linear algebraic system for finding some numbers that define the solution to problem (11)-(13) with $M > 1$. Also in Section 3 we

derive an integral equation for the effective (self-consistent) field in the medium with $M(a) \to \infty$ cylinders as $a \to 0$. At the end of Section 3 these results are applied to the problem of changing the refraction coefficient of a given material by embedding many thin perfectly conducting cylinders into it.

In Section 4 conclusions are formulated.

In Appendix formulas (14)-(15) are derived.

2 EM wave scattering by one thin perfectly conducting cylinder

Consider problem (11)-(13) with $\Omega = D_1$, Ω' being the complement to D_1 in $\mathbb{R}^2$. Assume for simplicity that D_1 is a circle $x_1^2 + x_2^2 \leq a^2$.

Let us look for a solution of the form

$$u = e^{i\kappa y} + \int_{S_1} g(\hat{x}, t)\sigma(t)dt, \qquad g(\hat{x}, t) := \frac{i}{4} H_0^{(1)}(\kappa|\hat{x} - t|), \tag{16}$$

where S_1 is the boundary of D_1, $H_0^{(1)}$ is the Hankel function of order 1, with index 0, and σ is to be found from the boundary condition (12). It is known (see, e.g., [5]) that

$$g(\kappa r) = \alpha(\kappa) + \frac{1}{2\pi} \ln \frac{1}{r} + o(1), \quad \text{as} \quad r \to 0, \tag{17}$$

where

$$\alpha(\kappa) := \frac{i}{4} + \frac{1}{2\pi} \ln \frac{2}{\kappa}. \tag{18}$$

The function (16) satisfies equations (11) and (13) for any σ, and if σ is such that function (16) satisfies boundary condition (12), then u solves problem (11)-(13). We assume σ sufficiently smooth (Hölder-continuous is sufficient).

The solution to problem (11)-(13) is known to be unique (see, e.g., [4]). Boundary condition (12) yields

$$-u_0(s) = \alpha(\kappa)Q + \int_{S_1} g_0(s, t)\sigma(t)dt, \qquad Q := \int_{S_1} \sigma(t)dt, \tag{19}$$

$$u_0(s) := e^{i\kappa s_2}, \ s \in S_1; \ g_0(s, t) = \frac{1}{2\pi} \ln \frac{1}{r_{st}}, \ r_{st} := |s - t|. \tag{20}$$

If $ka \ll 1$, $k^2 = \kappa^2 + k_3^2$, then

$$u_0(s) = 1 + O(\kappa a).$$

Equation (19) is uniquely solvable for σ if a is sufficiently small [5].

We are interested in finding asymptotics of Q as $a \to 0$, because $u(\hat{x})$ in (16) can be well approximated in the region $|\hat{x}| \gg a$ by the formula

$$\begin{aligned} u(\hat{x}) &= u_0(\hat{x}) + g(\hat{x}, 0)Q + o(1), \quad a \to 0, \\ u_0(\hat{x}) &= e^{i\kappa x_2}, \quad x_2 = y. \end{aligned} \tag{21}$$

To find asymptotics of Q as $a \to 0$, let us integrate equation (19) over S_1 and obtain

$$-u_0(0)|S_1| = \alpha(\kappa)Q|S_1| - \int_{S_1} dt\sigma(t)\frac{1}{2\pi}\int_{S_1} \ln r_{st} ds, \tag{22}$$

where $|S_1|$ is the length of S_1, $|S_1| = 2\pi a$ if S_1 is the circle $|\hat{x}| = a$, and $r_{st} = |s - t|$. Denote

$$I := \frac{1}{2\pi}\int_{S_1} \ln r_{st} dt = O(a \ln a), \quad a \to 0. \tag{23}$$

If S_1 is the circle $|\hat{x}| = a$, integral (23) can be calculated analytically:

$$\begin{aligned} I &= \frac{a}{2\pi}\int_0^{2\pi} \ln\sqrt{2a^2 - 2a^2\cos(\psi - \varphi)} d\varphi \\ &= \frac{a}{2\pi}\int_0^{2\pi} \ln\sqrt{2a^2} d\varphi + \frac{a}{2\pi}\int_0^{2\pi} \ln\sqrt{2\sin^2\frac{\psi - \varphi}{2}} d\varphi. \end{aligned} \tag{24}$$

Thus,

$$I = a\ln(\sqrt{2}a) + \frac{a\ln 2}{2} + \frac{a}{2\pi}\int_0^{2\pi} \ln|\sin\frac{\psi - \varphi}{2}| d\varphi. \tag{25}$$

One can derive that

$$\int_0^{2\pi} \ln|\sin\frac{\psi - \varphi}{2}| d\varphi = 2\int_0^{\pi} \ln|\sin\theta| d\theta = -2\pi\ln 2. \tag{26}$$

Indeed, if $J := \int_0^\pi \ln|\sin\theta|d\theta$, then

$$J = \int_0^\pi \ln 2|\sin\frac{\theta}{2}\cos\frac{\theta}{2}|d\theta = \pi\ln 2 + \int_0^\pi \ln\sin\frac{\theta}{2}d\theta + \int_0^\pi \ln|\cos\frac{\theta}{2}|d\theta = \pi\ln 2 + 2J.$$

Thus, $J = -\pi\ln 2$. From (24) and (26) one gets

$$I = a\ln a(1 + O\left(\frac{1}{|\ln a|}\right), \quad a \to 0. \tag{27}$$

From (22) and (27) it follows that

$$Q = -\frac{2\pi u_0(0)}{\ln\frac{1}{a}}[1 + O\left(\frac{1}{|\ln a|}\right)], \quad a \to 0. \tag{28}$$

Therefore, the asymptotic solution to the scattering problem (11)-(13) in the case of one circular cylinder of radius a, as $a \to 0$, is

$$u(\hat{x}) \sim u_0(\hat{x}) - \frac{2\pi}{\ln\frac{1}{a}} g(\hat{x}, 0)u_0(0), \;\; a \to 0, \;\; |\hat{x}| > a. \tag{29}$$

Electromagnetic wave, scattered by the single cylinder, is calculated by formulas (14)-(15) in which $u = u(\hat{x}) := u(x_1, x_2)$ is given by formula (29).

3 Wave scattering by many thin cylinders

Problem (11)-(13) should be solved when Ω is a union of many small domains D_m, $\Omega = \cup_{m=1}^{M} D_m$. We assume that D_m is a circle of radius a centered at the point $\hat{x}_m$.

Let us look for u of the form

$$u(\hat{x}) = u_0(\hat{x}) + \sum_{m=1}^{M} \int_{S_m} g(\hat{x}, t)\sigma_m(t)dt. \tag{30}$$

We assume that the points $\hat{x}_m$ are distributed in a bounded domain D on the plane $P = xoy$ by formula (2). The field $u_0(\hat{x})$ is the same as in Section 2, $u_0(\hat{x}) = e^{i\kappa y}$, and Green's function g is the same as in formulas (16)-(18). It follows from (2) that $M = M(a) = O(\ln \frac{1}{a})$. We define the effective field, acting on the D_j by the formula

$$u_e = u_e^{(j)} = u(\hat{x}) - \int_{S_j} g(\hat{x}, t)\sigma_j(t)dt, \quad |\hat{x} - \hat{x}_j| > a, \tag{31}$$

which can also be written as

$$u_e(\hat{x}) = u_0(\hat{x}) + \sum_{m=1, m\neq j}^{M} \int_{S_m} g(\hat{x}, t)\sigma_m(t)dt.$$

We assume that the distance $d = d(a)$ between neighboring cylinders is much greater than a:

$$d \gg a, \quad \lim_{a\to 0} \frac{a}{d(a)} = 0. \tag{32}$$

Let us rewrite (30) as

$$u = u_0 + \sum_{m=1}^{M} g(\hat{x}, \hat{x}_m)Q_m + \sum_{m=1}^{M} \int_{S_m} [g(\hat{x}, t) - g(\hat{x}, \hat{x}_m)]\sigma_m(t)dt, \tag{33}$$

where

$$Q_m := \int_{S_m} \sigma_m(t)dt. \tag{34}$$

As $a \to 0$, the second sum in (33) (let us denote it Σ_2) is negligible compared with the first sum in (33), denoted Σ_1,

$$|\Sigma_2| \ll |\Sigma_1|, \quad a \to 0. \tag{35}$$

The proof of this is similar to the one given in [6] for a similar problem in $\mathbb{R}^3$.

Let us check that

$$|g(\hat{x}, \hat{x}_m)Q_m| \gg |\int_{S_m} [g(\hat{x}, t) - g(\hat{x}, x_m)]\sigma_m(t)dt|, \quad a \to 0. \tag{36}$$

If $k|\hat{x}-\hat{x}_m| \gg 1$, and $k>0$ is fixed then

$$|g(\hat{x},\hat{x}_m)| = O(\frac{1}{|\hat{x}-\hat{x}_m|^{1/2}}), \qquad |g(\hat{x},t)-g(\hat{x},x_m)| = O(\frac{a}{|\hat{x}-\hat{x}_m|^{1/2}}),$$

and $Q_m \neq 0$, so estimate (36) holds.

If

$$|\hat{x}-\hat{x}_m| \sim d \gg a,$$

then

$$|g(\hat{x},\hat{x}_m)| = O(\frac{1}{ln\frac{1}{a}}), \qquad |g(\hat{x},t)-g(\hat{x},x_m)| = O(\frac{a}{d}),$$

as follows from the asymptotics of $H_0^1(r) = O(\ln\frac{1}{r})$ as $r \to 0$, and from the formulas $\frac{dH_0^1(r)}{dr} = -H_1^1(r) = O(\frac{1}{r})$ as $r \to 0$. Thus, (36) holds for $|\hat{x}-\hat{x}_m| \gg d \gg a$.

Consequently, the scattering problem is reduced to finding numbers Q_m, $1 \leq m \leq M$.

Let us estimate Q_m asymptotically, as $a \to 0$. To do this, we use the exact boundary condition on S_m, which yields

$$-u_e(s) = \int_{S_j} g(s,t)\sigma_j(t)dt, \quad s \in S_j. \tag{37}$$

The function $u_e(s)$ is twice differentiable, so

$$u_e(s) = u_e(\hat{x}_j)(1+O(ka)).$$

Neglecting the term $O(ka)$ as $a \to 0$, rewrite equation (37) as

$$-u_e(\hat{x}_j) = \int_{S_j} g(s,t)\sigma_j(t)dt. \tag{38}$$

This equation is similar to (19): the role of $u_0(0)$ is played by $u_e(x_j)$. Repeating the argument, given in Section 2, one obtains a formula, similar to (28):

$$Q_j = -\frac{2\pi u_e(\hat{x}_j)}{\ln\frac{1}{a}}[1+o(1)], \quad a \to 0. \tag{39}$$

Formula, similar to (29), is

$$u(\hat{x}) \sim u_0(\hat{x}) - \frac{2\pi}{\ln\frac{1}{a}} \sum_{m=1}^{M} g(\hat{x},\hat{x}_m)u_e(\hat{x}_m), \quad a \to 0. \tag{40}$$

The numbers $u_e(\hat{x}_m)$, $1 \leq m \leq M$, in (40) are not known. Setting $\hat{x} = \hat{x}_j$ in (40), neglecting $o(1)$ term, and using the definition (31) of the effective field, one gets a linear algebraic system for finding numbers $u_e(\hat{x}_m)$:

$$u_e(\hat{x}_j) = u_0(\hat{x}_j) - \frac{2\pi}{\ln\frac{1}{a}} \sum_{m\neq j} g(\hat{x}_j,\hat{x}_m)u_e(\hat{x}_m), \quad 1 \leq j \leq M. \tag{41}$$

This system can be solved numerically if the number M is not very large, say $M \leq O(10^3)$.

If M is very large, $M = M(a) \to \infty,\ a \to 0$, then we derive a linear integral equation for the limiting effective field in the medium obtained by embedding many cylinders.

Passing to the limit $a \to 0$ in system (41) is done as in [13]. Consider a partition of the domain D into a union of $\mathbf{P}$ small squares Δ_p, of size $b = b(a)$, $b \gg d \gg a$. For example, one may choose $b = O(a^{1/4})$, $d = O(a^{1/2})$, so that there are many discs D_m in the square Δ_p. We assume that squares Δ_p and Δ_q do not have common interior points if $p \neq q$. Let $\hat{y}_p$ be the center of Δ_p. One can also choose as $\hat{y}_p$ any point $\hat{x}_m$ in a domain $D_m \subset \Delta_p$. Since u_e is a continuous function, one may approximate $u_e(\hat{x}_m)$ by $u_e(\hat{y}_p)$, provided that $\hat{x}_m \subset \Delta_p$. The error of this approximation is $o(1)$ as $a \to 0$. Let us rewrite the sum in (41) as follows:

$$\frac{2\pi}{\ln\frac{1}{a}} \sum_{m \neq j} g(\hat{x}_j, \hat{x}_m) u_e(\hat{x}_m) = 2\pi \sum_{\substack{p=1 \\ x_j \notin \Delta_p}}^{\mathbf{P}} g(\hat{x}_j, \hat{y}_p) u_e(\hat{y}_p) \frac{1}{\ln\frac{1}{a}} \sum_{x_m \in \Delta_p} 1, \tag{42}$$

and use formula (2) in the form

$$\frac{1}{\ln\frac{1}{a}} \sum_{x_m \in \Delta_p} 1 = N(\hat{y}_p)|\Delta_p|[1 + o(1)]. \tag{43}$$

Here $|\Delta_p|$ is the volume of the square Δ_p.

From (42) and (43) one obtains:

$$\frac{2\pi}{\ln\frac{1}{a}} \sum_{m \neq j} g(\hat{x}_j, \hat{x}_m) u_e(\hat{x}_m) = 2\pi \sum_{\substack{p=1 \\ \hat{x}_j \notin \Delta_p}}^{\mathbf{P}} g(\hat{x}_j, \hat{y}_p) N(\hat{y}_p) u_e(\hat{y}_p)|\Delta_p|[1 + o(1)]. \tag{44}$$

The sum in the right-hand side in (44) is the Riemannian sum for the integral

$$\lim_{a \to 0} \sum_{p=1}^{\mathbf{P}} g(\hat{x}_j, \hat{y}_p) N(\hat{y}_p) u_e(\hat{y}_p)|\Delta_p| = \int_D g(\hat{x}, \hat{y}) N(\hat{y}) u(\hat{y}) dy, \quad u(\hat{x}) = \lim_{a \to 0} u_e(\hat{x}). \tag{45}$$

Therefore, system (41) in the limit $a \to 0$ yields the integral equation for the limiting effective field

$$u(\hat{x}) = u_0(\hat{x}) - 2\pi \int_D g(\hat{x}, \hat{y}) N(\hat{y}) u(\hat{y}) d\hat{y}. \tag{46}$$

One obtains system (41) if one solves equation (46) by a collocation method. Convergence of this method to the unique solution of equation (46) is proved in [10]. Existence and uniqueness of the solution to equation (46) are proved as in [6], where a three-dimensional analog of this equation was studied.

Applying the operator $\Delta_2 + \kappa^2$ to equation (46) yields the following differential equation for $u(\hat{x})$:

$$\Delta_2 u(\hat{x}) + \kappa^2 u(\hat{x}) - 2\pi N(\hat{x}) u(\hat{x}) = 0. \tag{47}$$

This is a Schrödinger-type equation, and $u(\hat{x})$ is its scattering solution corresponding to the incident wave $u_0 = e^{i\kappa y}$.

Let us assume that $N(x) = N$ is a constant. One concludes from (47) that the limiting medium, obtained by embedding many perfectly conducting circular cylinders, has new parameter $\kappa_N^2 := \kappa^2 - 2\pi N$. This means that $k^2 = \kappa^2 + k_3^2$ is replaced by $\tilde{k}^2 := k^2 - 2\pi N$. The quantity k_3^2 is not changed. One has $\tilde{k}^2 = \omega^2 n^2$, $k^2 = \omega^2 n_0^2$. Consequently, $n^2/n_0^2 = (k^2 - 2\pi N)/k^2$. Therefore, the new refraction coefficient n^2 is

$$n^2 = n_0^2(1 - 2\pi N k^{-2}), \tag{48}$$

Since the number $N > 0$ is at our disposal, equation (48) shows that choosing suitable N one can create a medium with a smaller, than n_0^2, refraction coefficient.

In practice one does not go to the limit $a \to 0$, but chooses a sufficiently small a. As a result, one obtains a medium with a refraction coefficient n_a^2, which differs from (48) a little, $\lim_{a\to 0} n_a^2 = n^2$.

4 Conclusions

Asymptotic, as $a \to 0$, solution is given of the EM wave scattering problem by many perfectly conducting parallel cylinders of radius a. The equation for the effective field in the limiting medium obtained when $a \to 0$ and the distribution of the embedded cylinders is given by formula (2). The presented theory gives formula (48) for the refraction coefficient in the limiting medium. This formula shows how the distribution of the cylinders influences the refraction coefficient.

5 Appendix

Let us derive formulas (14)-(15). Look for the solution to (3)-(4) of the form:

$$E_1 = e^{ik_3 z}\tilde{E}_1(x,y), \;\; E_2 = e^{ik_3 z}\tilde{E}_2(x,y), \;\; E_3 = e^{ik_3 z}u(x,y), \tag{49}$$

$$H_1 = e^{ik_3 z}\tilde{H}_1(x,y), \;\; H_2 = e^{ik_3 z}\tilde{H}_2(x,y), \;\; H_3 = 0, \tag{50}$$

where $k_3 = const$. Equation (3) yields

$$u_y - ik_3\tilde{E}_2 = i\omega\mu\tilde{H}_1, \;\; -u_x + ik_3\tilde{E}_1 = i\omega\mu\tilde{H}_2, \;\; \tilde{E}_{2,x} = \tilde{E}_{1,y}, \tag{51}$$

where, e.g., $\tilde{E}_{j,x} := \frac{\partial \tilde{E}_j}{\partial x}$. Equation (4) yields

$$ik_3\tilde{H}_2 = i\omega\epsilon\tilde{E}_1, \;\; ik_3\tilde{H}_1 = -i\omega\epsilon\tilde{E}_2, \;\; \tilde{H}_{2,x} - \tilde{H}_{1,y} = -i\omega\epsilon u. \tag{52}$$

Excluding $\tilde{H}_j$, $j = 1, 2$, from (51) and using (52), one gets

$$\tilde{E}_1 = \frac{ik_3}{\kappa^2} u_x, \ \ \tilde{E}_2 = \frac{ik_3}{\kappa^2} u_y, \ \ \tilde{E}_3 = u, \tag{53}$$

$$\tilde{H}_1 = \frac{k^2 u_y}{i\omega\mu\kappa^2}, \ \ \tilde{H}_2 = -\frac{k^2 u_x}{i\omega\mu\kappa^2} u_x, \ \ \tilde{H}_3 = 0. \tag{54}$$

Since $E_j = \tilde{E}_j e^{ik_3 z}$ and $H_j = \tilde{H}_j e^{ik_3 z}$, formulas (14)-(15) follow immediately from (53)-(54).

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